Measurement of Particle Accelerations with the Laser Doppler Technique

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Abstract An extension of the laser Doppler technique for measuring particle acceleration is presented. The basic principles of the technique follow closely those introduced in [11], although numerous improvements have been implemented in the signal processing for increasing the reliability of individual estimates of particle acceleration. The main contribution of this study is to identify and quantify the errors due to optical fringe divergence in the detection volume of the present laser Doppler system, to introduce an appropriate experiment involving a falling wire and to compare the acceleration measurements of the laser Doppler system to the results of a particle tracking system with high-speed cameras in a highly turbulent flow. Noteworthy is the fact that all measurements were performed with a commercial off-the-shelf laser Doppler system.

1 Introduction

The Lagrangian acceleration yields deep insights into turbulence, because the fundamental conservation equations of fluid mechanics are cast in terms of this acceleration. The measurement of particle accelerations allows both the measurement of

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Lagrangian acceleration of the flow with the aid of small passive tracer particles as well as the Lagrangian acceleration of inertial particles.

The methods used to date were particle tracking velocimetry (PTV) with high-speed cameras [15] or silicon strip detectors [8, 16, 17]. Further measurements of Lagrangian and Eulerian accelerations have been performed with particle image velocimetry (PIV) [2, 3, 4, 5, 6, 7, 12, 18]. However, the laser Doppler technique can also be used to measure fluid acceleration, as demonstrated in [9, 10, 11], offering the advantages of a smaller measurement volume with high spatial and temporal resolution. Therefore, it is of principle interest to quantify the accuracy of this method and to compare it to other alternative methods. The big challenge is to use a commercial system and replace the usual signal processing by one, which extracts the particle velocity and acceleration instead of only the velocity.

The measurement volume of a usual laser Doppler system has a diameter of the order of $100\,\mu\mathrm{m}$. A particle crossing the measurement volume with $10\,\mathrm{m/s}$ needs $10\,\mu\mathrm{s}$ to pass the volume. Assuming a particle acceleration of $1000\,\mathrm{m/s}^2$, the change of the particle velocity while passing the measurement volume is only $0.01\,\mathrm{m/s}$ which is a relative change of $0.1\,\%$. To resolve this small change both the optical system and the signal processing must be highly accurate.

This paper introduces the technique, the optical alignment, the signal processing and test measurements achieving an acceptable accuracy with an off-the-shelf commercial laser Doppler system. The test measurements yield yield reliable information about the possible accuracy of a given optical system.

2 Measurement system

One aim of this investigation was to achieve acceleration measurements with a commercial laser Doppler system with no customized or additional components, simultaneously demonstrating the possibility of employing existing laser Doppler systems for such measurements, while only the signal processing must be modified.

The optical setup used here is a standard, three-velocity component laser Doppler system from TSI, although only one component was used throughout this study. The transmitting lens had a focal length of $363\,\mathrm{mm}$. The measurement volume has a diameter of $60\,\mu\mathrm{m}$, a length of $500\,\mu\mathrm{m}$ and a fringe distance of $3.73\,\mu\mathrm{m}$. The standard signal processor provided the frequency shift signal of $40\,\mathrm{MHz}$ for the Bragg cell, but was otherwise only used for down-mixing and signal conditioning (phase-conserving amplification and $20\,\mathrm{MHz}$ high-pass filter to remove the pedestal DC-component) before digital acquisition of the signals with a high-speed digital scope card. To extract the mean velocity and the particle acceleration from the laser Doppler signals, a special post-processing has been developed.

3 Signal processing

Once a laser Doppler signal is digitized, the signal processing can estimate the (mean) velocity and the particle acceleration. The corresponding signal quantities are the instantaneous frequency and its change over the signal length. Several signal processing methods have been developed, investigated and discussed in the past [11, 13, 14]. Model-based estimators minimizing the L_2 norm are a common tool to achieve highest accuracy. In that case, the signal envelope must be known a priori or estimated by the procedure. However, in this application, the envelope may strongly fluctuate in amplitude. Only the high-frequency modulation provides reliable information.

Therefore, a robust signal processing as introduced in [14] is preferred. As a signal model, this method uses a complex, analytic, harmonic signal of constant amplitude

$$s_i = \cos(\pi\gamma(t_i - T)^2 + 2\pi f_{\mathrm{D}}(t_i - T)) + \mathrm{j}\sin(\pi\gamma(t_i - T)^2 + 2\pi f_{\mathrm{D}}(t_i - T)) \quad (1)$$

with the particle arrival time T, the sampling times t_i, the mid-point frequency f_{D}, the frequency gradient γ and the imaginary unit j. This complex model signal s is correlated with the measured (real) signal $\hat{s}$.

$$R = \sum_i s_i \hat{s}_i \quad (2)$$

The two parameters of the model, the Doppler frequency f_{D} corresponding to the particle velocity, and γ corresponding to the acceleration are iteratively optimized such that the absolute value of the correlation $|R|$ is maximized. The advantage of this procedure compared to an estimator minimizing the L_2 norm is that it is almost independent of the signals envelope. Nonetheless, it is very efficient, reaching almost the Cramér Rao lower bound. Furthermore, the estimator is almost independent of the absolute phase of the signal. Therefore, it is not necessary to consider or estimate the signal phase in the signal model.

4 Fringe distortions

The beam waists have been aligned carefully with the beam intersection point to avoid distortions of the fringe system as shown in Fig. 1. There are two different types of deformations caused by possible misalignments of the two beam waists. If the beam waists lie outside the intersection point, but both on the same side and at the same distance relative to the intersection point (Fig. 1b), the fringe spacing changes in the direction of the optical axis, while the spacing between fringes in the measurement direction remains almost constant. The fringe spacing becomes additionally inhomogeneous in the measurement direction if the beam waists are unequally located outside of the intersection point (Fig. 1c).

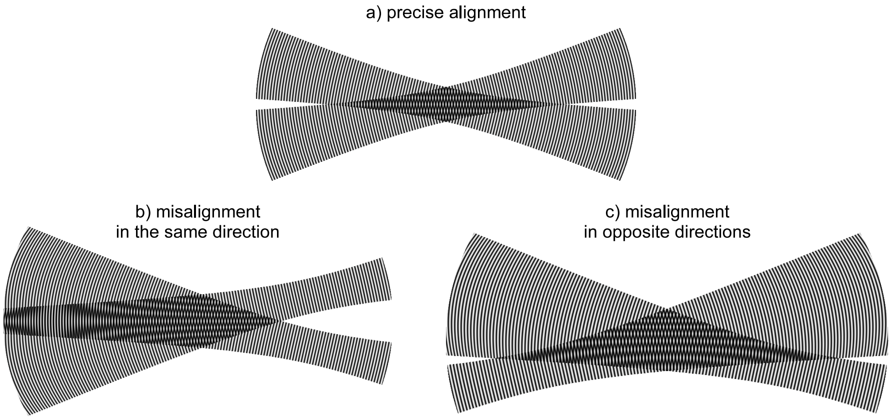

Fig. 1 Possible distortions of the fringe system due to misalignments of the beam waists (strongly emphasized

With a dual scanning slit beam profiler, the position of each beam waist could be adjusted along the beam with an accuracy of about ±0.5 mm by adjusting the distance between the transmitting fibers and the collimation lenses of the LDV probe. Following the expression given for the local spacing between fringes in [1] (Eq. 7.132), this leads to a (mean) fringe spacing of $\Delta x = 3.73\,\mu\mathrm{m}$ for the present optical system with a maximum fringe divergence of about 0.05 %, which can fulfill the requirement of 0.1 %. To investigate the actual fringe divergence in the measurement volume and systematic errors of the measurement system, a special validation experiment was designed, consisting of a thin wire falling through the volume.

5 Reference measurements

To evaluate the achievable accuracy of the measurement system, a reference experiment has been performed with a falling wire (Fig. 2), which has the acceleration of gravity while falling through the measurement volume. Any deviation from this value corresponds to a systematic error caused by optical misalignments yielding fringe distortions.

The measurement volume is scanned with the falling wire in the measurement direction of the laser Doppler system at different locations along the optical axis (vertical arrows in Fig. 3). A particle crossing a measurement volume with a misalignment of the beam waists as in Fig. 1c will see a change in the fringe spacing during its passage, causing a virtual acceleration and strong systematic errors. On the other hand, distortions as in Fig. 1b will only lead to different velocity measurements at different positions along the optical axis and no additional virtual acceleration. Unfortunately, the velocity depends on the falling height, which cannot

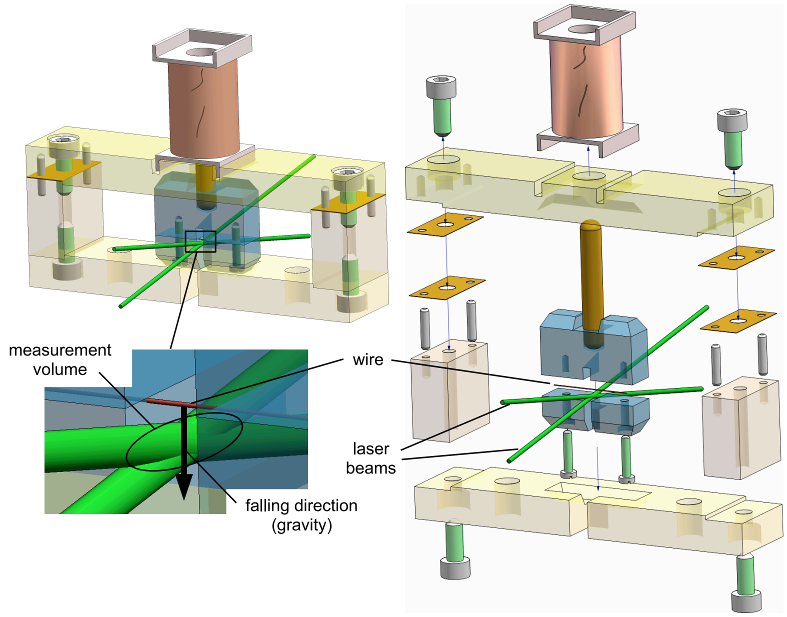

Fig. 2 Sketch of the falling wire experiment

be controlled with the required accuracy. Therefore, the measurement volume is rotated, so that the falling wire scans the measurement volume at two lines at an angle of ±17° about the optical axis. In that case, also this type of distortions will lead to changes of fringe spacing along the scan line and, therefore, significant virtual accelerations and deviations from the acceleration of gravity expected at this angle.

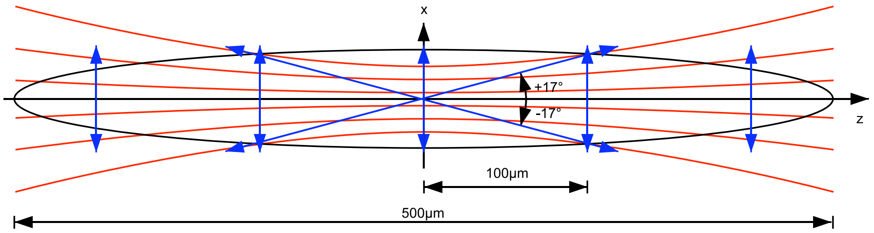

Fig. 3 LDV fringes (lines), measurement volume (ellipse) and scan lines (5 vertical and 2 tilted arrows) of the falling wire experiment

The results in Fig. 4 show that the optical system has almost no systematic errors within the measurement volume.

However, the particle velocity influences both the change of frequency due to fringe distortions and the signal duration. Therefore, systematic errors caused by fringe distortions scale with the square of the velocity. A conservative (worst case)

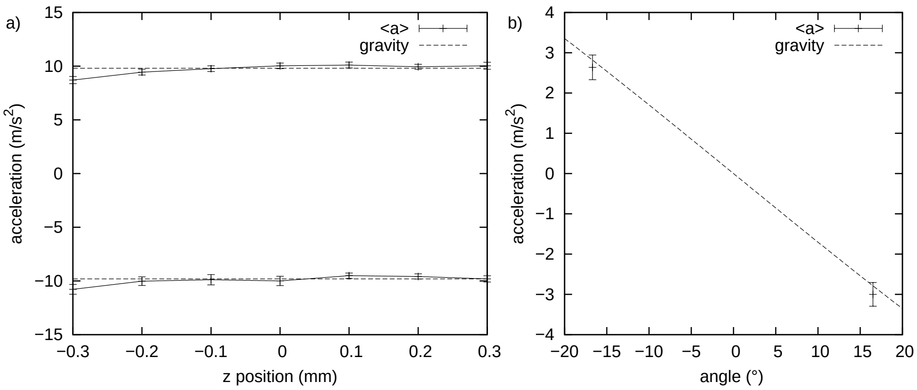

Fig. 4 Reference measurements for scanlines at a) ±90° and b) ±17°

estimation of the error bounds from the measurements above yields about 300 $\frac{\mathrm{m/s^2}}{\mathrm{(m/s)^2}}$ (or m^{-1}) derived from an error of $0.18\,\mathrm{m/s^2}$ at a velocity of $0.025\,\mathrm{m/s}$ for the measurement at $+17^\circ$.

6 Test measurement

A test measurement has been done in a turbulent flow, comparing the results of the laser Doppler system with the results obtained with a high-speed particle tracking system. The Lagrangian Exploration Module (Fig. 5) generates nearly homogeneous and isotropic turbulence with small mean flow [19]. The apparatus is shaped as an icosahedron containing 140 liters of water. The flow is driven by 12 independently controlled propellers, each at one of the vertices of the icosahedron. In Fig. 6 the normalized probability density function of the measured particle accelerations for both measurements at a Taylor micro scale Reynolds number of $R_\lambda \approx 195$ are shown. The acceleration values normalized with the standard deviation are obtained from the estimated acceleration values a_i with

$$a_{\mathrm{norm},i} = \frac{a_i - \bar{a}}{\sigma_a} \tag{3}$$

where $\bar{a}$ denotes the empiric mean value and σ_a the standard deviation. The two measurements nicely agree verifying that the laser Doppler system is a useful alternative to common techniques for measuring particle accelerations.

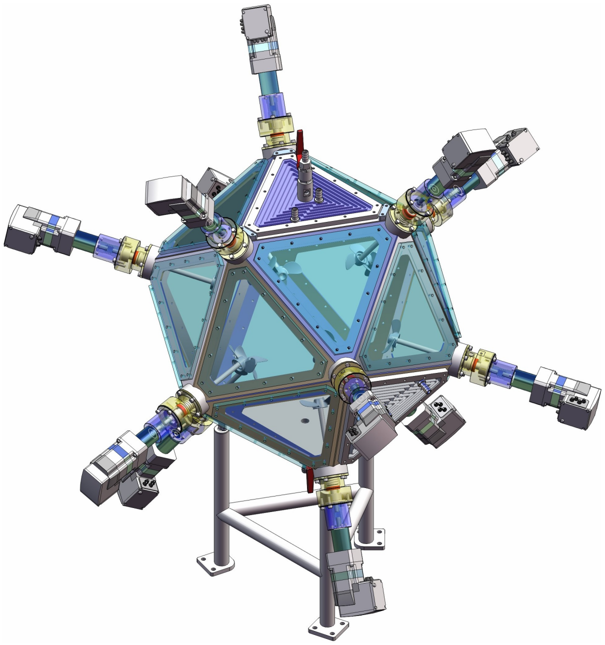

Fig. 5 Sketch of the Lagrangian Exploration Module

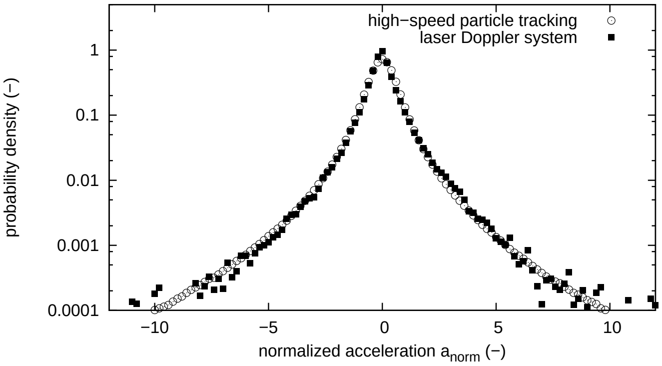

Fig. 6 Normalized probability density functions taken in a turbulent flow with two different measurement techniques

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